2316 **225**

Class–BA/B.Sc II (Sem. IV)
Subject –Mathematics
Paper-II Number Theory

Time Allowed : 3 Hrs **Maximum Marks :50**

Note :- Attempt any five questions selecting atleast Two from Each Section.

Section - A

1. (a) For any two integers a and b, with $a > 0$, there exists unique integers q and r such that $b = aq + r$, $0 \leq r < |a|$.

   (b) If m is an integer not divisible by 2 or 3 show that $24/m^2 + 23$. (6,4)

2. (a) Prove that $a^{2^n} + 1$ divides $a^{2^m} - 1$ if $m > 0$; & $n > 0$;

   also prove that $(a^{2^m} + 1, a^{2^n} + 1) = \begin{cases} 1 & \text{if } a \text{ is even} \\ 2 & \text{if } a \text{ is odd} \end{cases}$ for positive integers a, m,n.

   (b) Find L.C.M [714, 2030, 2205] (5,5)

3. (a) State and prove fundamental theorem of Arithmetic.

   (b) If p, q are primes such that $p - q = 2$, show that $p^p + q^q$ is divisible by $p + q$ i.e. $p^p + q^q$ is composite number. (5,5)

4. (a) If p_n is the nth prime, prove $p_n \leq 2^{2^{n-1}}$.

   (b) Show that $53^{103} + 103^{53}$ is divisible by 39. (5,5)

Section - B

5. (a) Solve 91 x $\equiv$ 1053 (mod 221)

(b) Show that $1^5+2^5+3^5+..........+100^5$ is divisible by 4. (5,

6. (a) Solve x $\equiv$ 5 (mod 11), x $\equiv$ 14 (mod 29) & x $\equiv$ 1 (mod 31) by using Chinese Reminder Theorem

(b) Show that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is always an integ $\forall\, n \in N$. (6,

7. (a) State and prove Wilson's Theorem.

(b) For any odd prime p, show that $2^2 . 4^2 ... 6^2 ....(p-1)^2 \equiv (-1)^{\frac{p+1}{2}}$ (mod p) usin Wilson's Theorem. (5,

8. (a) State and prove Euler's Theorem.

(b) Show that $a^{560} \equiv 1$ (mod 561) if gcd (a, 561) = 1 however 561 is not a prime. (6,

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